Trends in Health Informatics

www.thi.reapress.com

THI. Vol. 2, No. 1 (2025) 63–67.

Paper Type: Original Article

On Model for Analysis of Skeletal Muscle Contraction

Evgeny L. Pankratov*

Nizhny Novgorod State Agrotechnical University, 97 Gagarin avenue, Nizhny Novgorod, 603950, Russia; elp2004@mail.ru.

Citation:

Received: 03 July 2024 *Revised: 17 September 2024* *Accepted: 05 November 2024*	Pankratov, E. L. (2025). A model for analysis of skeletal muscle contraction. *Trends in health informatics, 2*(1), 63-67.

Abstract

In this study, we present a comprehensive model of skeletal muscle contraction that accounts for both its mechanical and deformation properties. Unlike conventional models that treat muscle solely as a contractile element, the proposed framework integrates active and passive fiber components, offering a more accurate representation of biomechanical behavior. The model captures the dynamic interplay between contractile forces and elastic deformations within the muscle, providing insights into how microscopic mechanisms influence macroscopic motion. An analytical approach is also developed to examine the contraction process, enabling detailed evaluation of stress–strain relationships, energy distribution, and mechanical efficiency under various loading and activation conditions. This framework further allows direct comparison between theoretical predictions and experimental observations, supporting model validation and parameter optimization. Results indicate that incorporating deformation properties significantly improves the predictive accuracy of muscle performance, particularly in high-strain scenarios or under variable activation levels. The proposed approach offers a robust tool for advancing research in biomechanics, rehabilitation engineering, and the development of bio-inspired actuators. It may facilitate the design of more effective experimental studies and computational simulations.

Keywords: Muscle contraction, Process model, Analytical approach for analysis.

1 | Introduction

The prognosis of muscle contraction is an important factor in the study of the physiological characteristics of human movement. Knowledge of the informative parameters of muscle mechanical properties is used in medicine to treat patients [1–5]. In sports, predicting human muscle movement helps coaches improve the effectiveness of training. The capabilities of modern models enable conducting research and introducing corrections to treatment and training methods directly during their implementation. In this paper, we propose a model for the analysis of skeletal muscle contraction that accounts for its deformation properties. We introduce an analytical approach for the analysis of the considered muscle contraction.

Corresponding Author: elp2004@mail.ru

https://doi.org/10.22105/thi.vi.29

2 | Literature Review

Skeletal muscle contraction has been extensively studied from both experimental and theoretical perspectives, leading to the development of a variety of mathematical and biomechanical models. Early classical models, such as Hill-type formulations, treated muscle as a combination of contractile and elastic elements, providing a simplified representation of its mechanical behavior [1], [2]. While these models captured basic force–length and force–velocity relationships, they often neglected the complex deformation properties and viscoelastic behavior inherent to muscle tissue.

Subsequent studies have aimed to address these limitations by introducing more sophisticated frameworks. Petrov [3] and Chernous and Shilko [4] proposed models that incorporate both active contraction and passive elastic properties, thereby improving the accuracy of stress–strain predictions. Similarly, Pankratov and Bulaeva [5] highlighted the importance of viscoelasticity in capturing transient responses of muscle fibers. At the same time, Marque [6] demonstrated the utility of analytical approaches for evaluating layered tissue dynamics in biomedical applications.

Recent advancements emphasize multi-scale and layered modeling strategies that account for local deformations and the structural heterogeneity of muscle. For example, models conceptualizing muscle as a network of elastic threads embedded in an elastic-viscous substrate have shown promise in representing both macroscopic contraction and microscopic fiber interactions. These approaches enable analytical examination of stress distribution, energy storage, and mechanical efficiency under varying loading conditions [7–9].

Building on this foundation, the present study introduces a model that explicitly incorporates muscle deformation properties and a corresponding analytical framework for contraction analysis. By doing so, it bridges classical and contemporary approaches, offering a versatile tool for predicting skeletal muscle behavior under diverse mechanical scenarios.

3 | Method of Solution

In this section, we consider the model of skeletal muscle contraction and analyze it. In the framework of the model under consideration, we assume that the muscle is a locally flat object with the structure "Elastic thread - elastic-viscous substrate": a set of parallel threads connected to an elastic-viscous substrate. We will assume that the effective layer of tissue with depth H is reduced. A linear law of distribution along the coordinate q of the component of the displacement field normal to the muscle surface is adopted.

$$U(y\ z\ t) = V(z\ t)[1 + \alpha(y\ z\ t)\cdot y/z], \quad (1)$$

where U (y,z,t) is the normal to the muscle surface component of the displacement vector field; V (z,t) is the movement of a fiber point along the Oy axis, spaced from the edge at a distance z; H is the depth of the effective layer of the substrate; y is the coordinate directed from the free surface of the muscle; z is the fiber axis coordinate; α is the empirical parameter that takes into account possible deviations of the system under consideration from ideality. The equation of transverse oscillations of a thread on an elastic-viscous substrate has the following form [6].

$$m\frac{\partial^2 V(z,t)}{\partial t^2} = \frac{\partial}{\partial z}\left[T(y,z,t)\frac{\partial V(z,t)}{\partial z}\right] - q(y,z,t), \quad (2)$$

where m is the mass of a unit of the thread; T (y,z,t) is the thread tension force; q (y,z,t) is the distributed shear force from the side of the substrate, directed against the axis y. Force q (y,z,t) is determined through the tension in the muscle, the substrate σ, multiplied by the effective width b: $q = \sigma$ b. As boundary conditions, *Eq. (2)* is supplemented by the conditions for fastening the thread.

$$V(0\ t) = 0, V(L\ t) = 0, \quad (3\text{-a})$$

where l is the effective thread length. Initial conditions for the function V (z,t) could be written as

$$V(z,0) = V_0, \quad \left.\frac{\partial V(z,t)}{\partial t}\right|_{t=0} = 0. \tag{3-b}$$

We solve *Eq. (2)* with *Conditions (3)* by the recently introduced method of functional corrections [7], [8]. In the framework of the approach, we transform the thread tension force T (y,z,t) to the following form

$$T(y\ z\ t) = T0[1 + \varepsilon{\cdot}g(y\ z\ t)], \tag{4}$$

where T0 is the average value of the considered force, 0≤ ε <1, |g (y,z,t)|≤ 1. We determine the Solution of *Eq. (2)* as the following power series.

$$V(z,t) = \sum_{i=0}^{\infty} \varepsilon^i V_i(z,t). \tag{5}$$

Substitution of the considered form of *Solution (5)* and *Relation (4)* into *Eq. (2)* and *Conditions (3),* as well as grouping of terms at equal powers of the parameter, gives a possibility to obtain equations for functions Vi (z,t), boundary and initial conditions for them in the following form:

$$m\frac{\partial^2 V_0(z,t)}{\partial t^2} = T_0\frac{\partial^2 V_0(z,t)}{\partial^2 z} - q(y,z,t), \tag{6-a}$$

$$m\frac{\partial^2 V_i(z,t)}{\partial t^2} = T_0\frac{\partial^2 V_i(z,t)}{\partial^2 z} + T_0\frac{\partial}{\partial z}\left\{g(y,z,t)\frac{\partial V_{i-1}(z,t)}{\partial z}\right\}, \quad i\geq 1, \tag{6-b}$$

$$V_i(0,t) = 0, V_i(L,t) = 0, \left.\frac{\partial V_i(z,t)}{\partial t}\right|_{t=0} = 0, \quad i\geq 0;\ V_0(z,0)=V_0,\ V_i(z,0)=0,\ i\geq 1. \tag{7}$$

Eqs. (6) with *Conditions (7)* were solved by the Fourier variable separation method [9]. The considered solutions could be presented in the following form:

$$V_0(z,t) = \frac{V_0 L}{\pi n}\sum_{n=0}^{\infty}\left[(-1)^n - 1\right]\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right) -$$
$$-\frac{T_0}{m}\sum_{n=0}^{\infty}\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right)\int_0^L q(y,z,t)\sin\left(\frac{\pi n z}{L}\right)dz, \tag{8}$$

$$V_i(z,t) = -\frac{\pi n}{L}\frac{T_0}{m}\sum_{n=0}^{\infty}\sin\left(\frac{\pi n z}{L}\right)\sin\left(\sqrt{\frac{T_0}{m}}\frac{t}{L}\right)\times$$
$$\times\int_0^L\left\{g(y,z,t)\frac{\partial V_{i-1}(z,t)}{\partial z}\right\}\cos\left(\frac{\pi n z}{L}\right)dz, \quad i\geq 1. \tag{9}$$

The spatio-temporal distribution of the movement of a fiber point along the Oy axis was analyzed analytically using the second-order approximation in the framework of the method of function corrections. The approximation is usually good enough for qualitative analysis and for obtaining some quantitative results. All obtained results have been checked against numerical simulation results.

4 | Discussion

In this section, we analyze the spatio-temporal distribution of fiber-point displacements along the Oy axis. *Fig. 1* shows typical dependences of the considered distribution on the coordinate during fiber compression for various values of the external force q. An increase in the curve number corresponds to an increase in the force under consideration. Stretching the fiber results in the opposite effect. A similar result was obtained when analyzing the change in fiber over time.

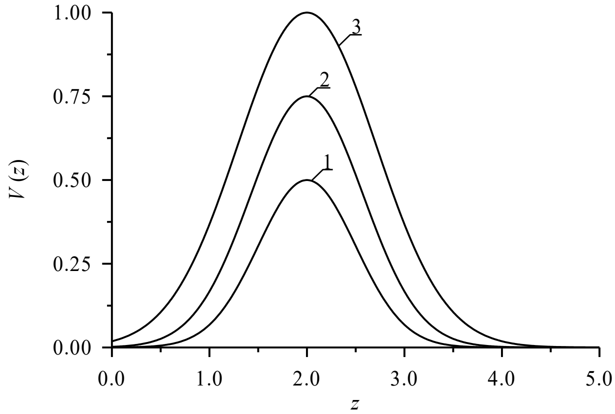

Fig. 1. Typical dependences of the distribution of the fiber point displacement along the Oy axis for various values of the external force q. An increase in the curve number corresponds to an increase in the considerate force.

5 | Conclusion

In this paper, we propose a model for analyzing skeletal muscle contraction that accounts for its deformation properties. We analyzed the considered model. We introduce an analytical approach for the analysis of the considered muscle contraction.

References

[1] Kiselev, I. N., Akberdin, I. R., Vertyshev, A. Y., Popov, D. V., & Kolpakov, F. A. (2019). A modular visual model of energy metabolism in human skeletal muscle. *Matematicheskaya biologiya i bioinformatika*, *14*(2), 373–392. https://doi.org/10.17537/2019.14.373

[2] Comincioli, V., & Naldi, G. (1990). Mathematical models in muscle contraction: Parallelism in the numerical approach. *Mathematical and computer modelling*, *13*(1), 109–115. https://doi.org/10.1016/0895-7177(90)90118-7

[3] Skrebenkov, E. A., & Vlasova, O. L. (2022). Mathematical simulation of efferent regulation of muscle contraction. *Biophysics*, *67*(2), 221–230. https://doi.org/10.1134/S0006350922020208

[4] Chernous, D. A., & Shilko, S. V. (2006). Modelling of contractive activity of the muscle tissue. *Russian journal of biomechanics*, *10*(3), 51–60. https://www.researchgate.net/profile/Serge-Shilko/publication/259821697_Modelling_of_Contractive_Activity_of_the_Muscle_Tissue/links/0f31752e0107088f98000000/Modelling-of-Contractive-Activity-of-the-Muscle-Tissue.pdf

[5] Pankratov, E. L., & Bulaeva, E. A. (2013). Doping of materials during manufacture p–n-junctions and bipolar transistors. Analytical Approaches to model technological approaches and ways of optimization of distributions of dopants. *Reviews in theoretical science*, *1*(1), 58–82. https://doi.org/10.1166/rits.2013.1004

[6] Marque, C., Duchene, J. M. G., Leclercq, S., Panczer, G. S., & Chaumont, J. (2007). Uterine EHG processing for obstetrical monitorng. *IEEE transactions on biomedical engineering*, (12), 1182–1187. https://doi.org/10.1109/TBME.1986.325698

[7] Zeng, W., Hume, D. R., Lu, Y., Fitzpatrick, C. K., Babcock, C., Myers, C. A., … & Shelburne, K. B. (2023). Modeling of active skeletal muscles: a 3D continuum approach incorporating multiple muscle

interactions. *Frontiers in bioengineering and biotechnology*, *11*, 1153692. https://doi.org/10.3389/fbioe.2023.1153692

[8] Loumeaud, A., Pouletaut, P., Bensamoun, S. F., George, D., & Chatelin, S. (2024). Multiscale mechanical modeling of skeletal muscle: A systemic review of the literature. *Journal of medical and biological engineering*, *44*(3), 337–356. https://doi.org/10.1007/s40846-024-00879-3

[9] Almonacid, J. A., Domínguez-Rivera, S. A., Konno, R. N., Nigam, N., Ross, S. A., Tam, C., & Wakeling, J. M. (2024). A three-dimensional model of skeletal muscle tissues. *SIAM journal on applied mathematics*, *84*(3), S538--S566. https://doi.org/10.1137/22M1506985